Comparison Principle of Very Weak Solutions for Nonhomogeneous Elliptic Equations

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Abstract: Under some suitable assumptions, a comparison principle of very weak solutions for quasi-linear elliptic equation $-\text{div}A(x, \nabla u) = f(x, u)$ is given by using McShane extension theorem to construct the Lipschitz continuous test function, and the corresponding results of some homogeneous equations are generalized.

Key words: Very weak solution; Comparison principle; Lipschitz continuous test function; McShane extension theorem

1. Introduction

In this paper, we study a comparison principle of very weak solutions to nonhomogeneous elliptic equation

$$-\text{div}A(x, \nabla u) = f(x, u).$$

(1.1)

In recent years, there exist many results of weak solutions for quasilinear elliptic equations. Gilbarg and Trudinger[1] established a comparison principle of classical solutions for second-order quasilinear elliptic equation. Tolksdorf[2] generalized the results of [1] and obtained a comparison principle of weak solutions for $\sum_{j=1}^{n} A_j(x, u, \nabla u) - A(x, u, \nabla u) = 0$. Damascelli[3] studied a comparison principle of weak solutions (in $W^{1, \infty}(\Omega)$) for $-\text{div} A(x, \nabla u) = g(x, u)$ by taking an appropriate test function. The right hand side of the equation (i.e. lower order term) in [1-3] satisfied non-increasing for $u$. The definition of very weak solutions for A-harmonic equations were given[4], Iwaniec et al.[5] obtained the existence and local integrability of very weak solutions to the A-harmonic equation by using Hodge decomposition method. By Hodge decomposition method to construct a proper test function, the uniqueness of the very weak solutions of $-\text{div} A(x, \nabla u) = f(x, u)$ is obtained under the condition of weak boundary value in [6]. More references about Hodge decomposition see [7-9].

Lewis[10] and ZHONG[11] studied the existence and uniqueness of very weak solutions for $D^{m}A(x, D^{m}u) = 0$ and $-\text{div} A(x, \nabla u) = \mu$ in Grand Sobolev space by using the method of maximal function to construct Lipschitz-type continuous test function. SHI[12]

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studied a comparison principle of very weak solutions to \(-\text{div}A(x, \nabla u) = 0\) by the similar method in [10-11]. ZHU\(^{[13]}\) introduced a comparison principle of very weak solutions for \(-\text{div}A(x, u, \nabla u) = f(x) + \text{div}(|\nabla u|^{p-2}\nabla u)\) by constructing a suitable test function, and combining Hardy-Littlewood maximum function et al.

However, the comparison principle of very weak solutions to equation (1.1) has not been studied yet. Inspired by [6,11-12], we study a comparison principle of equation (1.1) by using McShane extension theorem to contract the Lipschitz continuous test function. Compared with the right hand side in [13], our \(f(x, u)\) in (1.1) is more general, the reason is that the right hand side of [13] is actually \(f(x)\) which is independent of \(u\). In particular, owing to the appearance of \(f(x, u)\) in the proof, we apply Theorem 2.7 in [14] (i.e. Lemma 2.3 in this paper), Sobolev embedding theorem, Hölder’s and Young’s inequalities in order to estimate the integral term of \(f(x, u)\). In this paper, we assume \(f(x, u)\) to be non-increasing for \(u\), so that our result also holds in classical solution and weak solution cases.

Let \(\Omega \subset \mathbb{R}^n(n \geq 2)\) be a bounded open subset, \(1 < p < \infty\), the Carathéodory function \(A(x, \xi) : \Omega \times \mathbb{R}^N \to \mathbb{R}^n, f(x, \xi) : \mathbb{R} \times \mathbb{R}^n \to \mathbb{R}^n\), and \(f(x, \xi)\) is non-increasing for \(\xi\), \(A(x, \xi)\) and \(f(x, \xi)\) satisfies:

\[
\begin{align*}
\langle A(x, \xi_1) - A(x, \xi_2), \xi_1 - \xi_2 \rangle &\geq \alpha(\|\xi_1\| + \|\xi_2\|)^{p-2}\|\xi_1 - \xi_2\|^2, \\
|A(x, \xi_1) - A(x, \xi_2)| &\leq \beta(\|\xi_1\| + \|\xi_2\|)^{p-2}\|\xi_1 - \xi_2\|, \\
|f(x, \xi_1) - f(x, \xi_2)| &\leq \|\xi_1 - \xi_2\|^{p-1},
\end{align*}
\]

where \(x \in \Omega, \xi(i = 1, 2) \in \mathbb{R}^N, \zeta(i = 1, 2) \in \mathbb{R}^n, 0 < \alpha < \beta < \infty\).

**Remark 1** We know that \(f(x, u)\) is non-increasing for \(u\), this condition and above (1.4) are not contradict, for example, when \(u > 0\),

\[
\begin{cases}
  u^p, & 0 < u < 1 \\
  u^{1-p}, & u \geq 1.
\end{cases}
\]

**Definition 1.1** A function \(u(x) \in W^{1, r}(\Omega), \max\{1, p - 1\} < r < p\) is called a very weak solution to elliptic equation (1.1) if

\[
\int_{\Omega} \langle A(x, \nabla u), \nabla \phi \rangle \, dx = \int_{\Omega} (f(x, u), \phi) \, dx
\]

for all \(\phi \in C_0^\infty(\Omega)\).

2. **Preliminary Lemmas and Main Result**

In order to prove the main result, we need the following lemmas.

**Lemma 2.1**\(^{[9]}\) Let \(h \in W^{1,q}(\Omega), 1 < q < \infty\). Then there exists a set \(N \subset \mathbb{R}^n, |N| = 0\), such that

\[
|h(x) - h(y)| \leq c|x - y|(M(|Dh|)(x) + M(|Dh|)(y))
\]

for every \(x, y \in \mathbb{R}^n\setminus N\), where \(c = c(n) > 0\), \(Mh(x)\) is a Hardy-Littlewood maximal function of \(h(x)\).

Let \(h(x) \in W^{1, r}_0(\Omega), 1 < r < \infty\), extending \(h(x) = 0\) to \(\mathbb{R}^n\), so that \(h(x) = 0\) in \(C\Omega\). For a real number \(\lambda > 0\), we denote

\[
F_\lambda = \{x \in \Omega \setminus N : M(|Dh|)(x) \leq \lambda, |h(x)| \leq \lambda d(x, \partial \Omega)\}
\]

\[
= \{x \in \Omega \setminus N : g(x) \leq \lambda\},
\]

where \(g(x) = \max\{M(|Dh|)(x), |h(x)|d^{-1}(x, \Omega)\}\).
It is easy to prove that $h|_{F_\lambda \cup C \Omega}$ is $\lambda$-Lipschitz continuity, where the constant $C = C(n) \geq 1$. Applying McShane extension theorem, we get the following lemma:

**Lemma 2.2** Let $h(x) \in W^{1, \infty}_0(\Omega)$, for any given $\lambda > 0$, $F_\lambda$ was defined by Lemma 2.1, then there exists Lipschitz continuous function $h_\lambda$ satisfying:

(i) $h_\lambda(x) = h(x)$ for every $x \in F_\lambda$;
(ii) $|Dh_\lambda(x)| \leq C(n)\lambda$ for every $x \in \mathbb{R}^n$;
(iii) $h_\lambda(x) = 0$ where $x \in \partial \Omega$;
(iv) $\|Dh_\lambda(x)\|_\infty = \|Dh(x)\|_\infty$ for a.e. $x \in F_\lambda$.

**Lemma 2.3** Every function $u: U \rightarrow \mathbb{R}$ of class $C^{0,1}(U)$ belongs to $W^{1,\infty}_{\log}(U)$ (where $U$ is the open set in $\mathbb{R}^n$).

Our main result is the following comparison principle:

**Theorem 2.1** Assume that the equation (1.1) satisfies (1.2), (1.3) and (1.4). There exists a constant $0 < \varepsilon_0 = \varepsilon_0(n, p, \alpha, \beta, \gamma) < 1$ such that solutions $u_1, u_2 \in W^{1, \infty}_0(\Omega)$, where $r > p - \varepsilon_0$, if $u_1(x) \geq u_2(x)$ on $\partial \Omega$, then $u_1(x) \geq u_2(x)$ on $\Omega$.

**3. Proof of Theorem**

In the following proof, all the constants $C$ may change from line to line.

**Proof** Let $u_1, u_2 \in W^{1, \infty}_0(\Omega)$ are solutions of equation (1.1), and $u_1 \geq u_2$ on $\partial \Omega$. We consider $v(x) = \min\{0, u_1 - u_2\}$ and know that $v(x) \in W^{1, \infty}_0(\Omega)$. From Lemma 2.1 and Lemma 2.2, there exists $v_\lambda$ as Lipschitz continuous extension of function $v(x)$ on $F_\lambda \cup C \Omega$, and $v_\lambda$ can be used as a test function in Definition 1.1 because it satisfied (i)-(iv) from Lemma 2.2. Next we have

$$\int \langle A(x, \nabla u), \nabla v_\lambda \rangle dx = \int \langle f(x, u), v_\lambda \rangle dx.$$  

Due to $u_1, u_2$ are solutions of equation (1.1), we obtained

$$\int \langle A(x, \nabla u_1), \nabla v_\lambda \rangle dx = \int \langle f(x, u_1), v_\lambda \rangle dx,$$

$$\int \langle A(x, \nabla u_2), \nabla v_\lambda \rangle dx = \int \langle f(x, u_2), v_\lambda \rangle dx.$$  

The above two formulas are subtracted,

$$\int \langle A(x, \nabla u_1) - A(x, \nabla u_2), \nabla v_\lambda \rangle dx = \int \langle f(x, u_1) - f(x, u_2), v_\lambda \rangle dx.$$  

Note that $\Omega = \Omega_1 \cup \Omega_2 = \{x \in \Omega : u_1(x) \geq u_2(x)\} \cup \{x \in \Omega : u_1(x) < u_2(x)\}$. Theorem 2.1 is clearly in $\Omega_1$. In order to get our result, we only need to prove $\Omega_2$ is empty, that is $u_1(x) \geq u_2(x)$ on $\Omega_2$. There have

$$\int \langle A(x, \nabla u_1) - A(x, \nabla u_2), \nabla v_\lambda \rangle dx = \int \langle f(x, u_1) - f(x, u_2), v_\lambda \rangle dx. \quad (3.1)$$

Owing to Lemma 2.1, there have $v_\lambda = v$ on $F_\lambda$, so

$$\int \langle A(x, \nabla u_1) - A(x, \nabla u_2), \nabla u_1 - \nabla u_2 \rangle dx$$

$$= - \int_{\Omega_2 \cap F_\lambda} \langle A(x, \nabla u_1) - A(x, \nabla u_2), \nabla v_\lambda \rangle dx + \int_{F_\lambda} \langle f(x, u_1) - f(x, u_2), u_1 - u_2 \rangle dx$$

$$+ \int_{\Omega_2 \setminus F_\lambda} \langle f(x, u_1) - f(x, u_2), v_\lambda \rangle dx$$

\begin{equation}
\leq |I_1| + |I_2| + |I_3|.
\end{equation}

We combined (1.2) to get the following estimate,
\begin{equation}
\int_{F_\lambda} \langle A(x, \nabla u_1) - A(x, \nabla u_2), \nabla u_1 - \nabla u_2 \rangle dx \\
\geq \alpha \int_{F_\lambda} (|\nabla u_1| + |\nabla u_2|)^p - 2|\nabla u_1 - \nabla u_2|^2 dx.
\end{equation}

According to the condition (1.3) and (1.4), Lemma 2.2 and the Sobolev embedding theorem, we deduce that
\begin{align}
|I_1| & \leq C \beta \lambda \int_{\Omega \setminus F_\lambda} (|\nabla u_1| + |\nabla u_2|)^p - 2|\nabla u_1 - \nabla u_2| dx, \\
|I_2| & \leq \int_{F_\lambda} |u_1 - u_2|^p dx \leq C \int_{F_\lambda} |\nabla u_1 - \nabla u_2|^p dx.
\end{align}

Next, by using Hölder’s and Young’s inequalities, the Sobolev embedding theorem, Lemma 2.2 and Lemma 2.3, we have
\begin{align}
|I_3| & \leq \int_{\Omega \setminus F_\lambda} |u_1 - u_2|^{p-1} |v_\lambda| dx \\
& \leq \left( \int_{\Omega \setminus F_\lambda} |u_1 - u_2|^p dx \right)^{\frac{p-1}{p}} \left( \int_{\Omega \setminus F_\lambda} |v_\lambda|^p dx \right)^{\frac{1}{p}} \\
& \leq C(\varepsilon') \int_{\Omega \setminus F_\lambda} |u_1 - u_2|^p dx + \varepsilon' \int_{\Omega \setminus F_\lambda} |v_\lambda|^p dx \\
& \leq C(\varepsilon') C \int_{\Omega \setminus F_\lambda} |\nabla u_1 - \nabla u_2|^p dx + \varepsilon' C^p - 1 \int_{\Omega \setminus F_\lambda} \lambda dx,
\end{align}

where from the third line to the fourth line, we apply Lemma 2.3, since \( v_\lambda \in C^{0,1}(\Omega) \), \( v_\lambda \in W^{1,\infty}_\text{loc}(\Omega) \), that is \( v_\lambda \in W^{1,p}_\text{loc}(\Omega) \), \( 1 < p \leq +\infty \).

Combining (3.3) and the estimates of \(|I_1|, |I_2|, |I_3|\), there have
\begin{equation}
\begin{split}
\alpha \int_{F_\lambda} (|\nabla u_1| + |\nabla u_2|)^p - 2|\nabla u_1 - \nabla u_2|^2 dx \\
\leq & C \beta \lambda \int_{\Omega \setminus F_\lambda} (|\nabla u_1| + |\nabla u_2|)^p - 2|\nabla u_1 - \nabla u_2| dx \\
+ & C \int_{F_\lambda} |\nabla u_1 - \nabla u_2|^p dx + C(\varepsilon') C \int_{\Omega \setminus F_\lambda} |\nabla u_1 - \nabla u_2|^p dx + \varepsilon' C^p - 1 \int_{\Omega \setminus F_\lambda} \lambda dx.
\end{split}
\end{equation}

Multiplying both sides of the above inequality by \( \lambda^{-1-\varepsilon}(0 < \varepsilon < 1) \) and integrating \( \lambda \) on \( (0, +\infty) \), we deduce that
\begin{align}
& \frac{\alpha}{\varepsilon} \int_{\Omega_2} (|\nabla u_1| + |\nabla u_2|)^p - 2|\nabla u_1 - \nabla u_2|^2 g(x)^{-\varepsilon} dx \\
\leq & \frac{C \beta}{1 - \varepsilon} \int_{\Omega_2} (|\nabla u_1| + |\nabla u_2|)^p - 2|\nabla u_1 - \nabla u_2| g(x)^{1-\varepsilon} dx \\
+ & \frac{C}{\varepsilon} \int_{\Omega_2} |\nabla u_1 - \nabla u_2|^p g(x)^{-\varepsilon} dx + \frac{C(\varepsilon') C}{1 - \varepsilon} \int_{\Omega_2} |\nabla u_1 - \nabla u_2|^p g(x)^{1-\varepsilon} dx \\
+ & \frac{\varepsilon' C^p - 1}{(1 - \varepsilon)} \int_{\Omega_2} g(x)^{1-\varepsilon} dx \\
\leq & \frac{C \beta}{1 - \varepsilon} \int_{\Omega_2} (|\nabla u_1| + |\nabla u_2|)^p - 2|\nabla u_1 - \nabla u_2| g(x)^{1-\varepsilon} dx.
\end{align}
+ \frac{C(1-C(\varepsilon'))}{\varepsilon} \int_{\Omega_2} |\nabla u_1 - \nabla u_2|^p g(x)^{-\varepsilon} dx + \frac{\varepsilon'C\lambda^{-1}}{(1-\varepsilon)} \int_{\Omega_2} g(x)^{1-\varepsilon} dx \\
leq C\beta \frac{1}{1-\varepsilon} \int_{\Omega_2} (|\nabla u_1| + |\nabla u_2|)^{p-2} |\nabla u_1 - \nabla u_2| g(x)^{1-\varepsilon} dx + \frac{\varepsilon'C}{(1-\varepsilon)} \int_{\Omega_2} |g(x)|^{1-\varepsilon} dx \\
leq |I_4| + |I_5|. \tag{3.5} \\

\textbf{Remark 2} \quad \text{In (3.5), we need point out, since } \varepsilon' < 1 \text{ and } C(\varepsilon') > 1 \text{ are coefficients of Young's inequality, } C \text{ is a positive constant in Sobolev embedding theorem, then } \frac{C(1-C(\varepsilon'))}{\varepsilon} < 0 \text{ in the sixth line of (3.5). Meanwhile, } \lambda^{-1} \text{ can be merged into } C \text{ because } \lambda > 0. \\

Similar to [12], recalling the definition of } g(x), \text{ combing our test function, we deduce that} \\
|I_4| = \frac{C\beta}{1-\varepsilon} \int_{\Omega_2} (|\nabla u_1| + |\nabla u_2|)^{p-2} |\nabla u_1 - \nabla u_2| g(x)^{1-\varepsilon} dx \\
\leq \frac{C\beta}{1-\varepsilon} \int_{\Omega_2} (|\nabla u_1| + |\nabla u_2|)^{p-1} g(x)^{1-\varepsilon} dx \\
\leq C\beta \left( \int_{\Omega_2} (|\nabla u_1| + |\nabla u_2|)^{p-\varepsilon} dx \right)^{\frac{p-1}{p-\varepsilon}} \left( \int_{\Omega_2} g(x)^{p-\varepsilon} dx \right)^{\frac{1}{p-\varepsilon}} \\
\leq C\beta \left( \int_{\Omega_2} (|\nabla u_1| + |\nabla u_2|)^{p-\varepsilon} dx \right)^{\frac{p-1}{p-\varepsilon}} \left( \int_{\Omega_2} |\nabla u_1 - \nabla u_2|^{p-\varepsilon} dx \right)^{\frac{1}{p-\varepsilon}}. \tag{3.6} \\

\text{By using Hölder's and Young's inequalities, we obtain} \\
|I_5| = \frac{\varepsilon'C}{(1-\varepsilon)} \int_{\Omega_2} |g(x)|^{1-\varepsilon} dx \\
\leq \frac{\varepsilon'C}{(1-\varepsilon)} \left( \varepsilon'' \int_{\Omega_2} |g(x)|^{p-\varepsilon} dx + C(\varepsilon'')|\Omega_2| \right) \\
\leq \frac{\varepsilon'C}{(1-\varepsilon)} \left( \varepsilon'' \int_{\Omega_2} |\nabla u_1 - \nabla u_2|^{p-\varepsilon} dx + C(\varepsilon'')|\Omega_2| \right), \tag{3.7} \\
\text{where } \varepsilon'' \text{ and } C(\varepsilon'') \text{ stand for the small and large constants respectively from Young’s inequality.} \\

\text{Similar to [12], We distinguish the proof into two cases:} \\
\textbf{Case 1} \quad p \geq 2. \text{ In this case, by using Hölder’s inequality we have} \\
\int_{\Omega_2} |\nabla u_1 - \nabla u_2|^{p-\varepsilon} dx \\
\leq C \left[ \int_{\Omega_2} |\nabla u_1 - \nabla u_2|^{p} g(x)^{-\varepsilon} dx \right]^{\frac{p-\varepsilon}{p}} \left[ \int_{\Omega_2} g(x)^p dx \right]^{\frac{\varepsilon}{p}} \\
\leq C \left[ \int_{\Omega_2} |\nabla u_1 - \nabla u_2|^{p} g(x)^{-\varepsilon} dx \right]^{\frac{p-\varepsilon}{p}} \left[ \int_{\Omega_2} |\nabla u_1 - \nabla u_2|^{p-\varepsilon} dx \right]^{\frac{1}{p}}. \tag{3.8} \\
\text{This is easy to implies} \\
\int_{\Omega_2} |\nabla u_1 - \nabla u_2|^{p-\varepsilon} dx \\
\leq C \int_{\Omega_2} |\nabla u_1 - \nabla u_2|^{p} g(x)^{-\varepsilon} dx \\
\leq C \int_{\Omega_2} (|\nabla u_1| + |\nabla u_2|)^{p-2} |\nabla u_1 - \nabla u_2|^2 g(x)^{-\varepsilon} dx. \tag{3.9}
Combining (3.5), (3.6), (3.7), (3.8) and (3.9), we have
\[
\int_{\Omega_2} |\nabla u_1 - \nabla u_2|^{p-\varepsilon} \, d\mathbf{x}
\leq \frac{C\varepsilon}{1-\varepsilon} \left[ \int_{\Omega_2} (|\nabla u_1| + |\nabla u_2|)^{p-\varepsilon} \, d\mathbf{x} \right] \frac{\varepsilon}{p-\varepsilon}
\times \left[ \int_{\Omega_2} |\nabla u_1 - \nabla u_2|^{p-\varepsilon} \, d\mathbf{x} \right]^{\frac{1}{p-\varepsilon}}
+ \varepsilon^{\varepsilon} C \left( \varepsilon'' \int_{\Omega_2} |\nabla u_1 - \nabla u_2|^{p-\varepsilon} \, d\mathbf{x} + C(\varepsilon'')|\Omega_2| \right),
\]
where the above inequality holds for all 0 < \varepsilon \leq \varepsilon_0 < 1, and C doesn't depend on \varepsilon. We let \varepsilon \to 0, then we get \( u_1 = u_2 \) a.e. in \( \Omega_2 \) and Theorem 2.1 were proved.

**Case 2** \( 1 < p < 2 \). This implies
\[
\int_{\Omega_2} |\nabla u_1 - \nabla u_2|^{p-\varepsilon} \, d\mathbf{x}
= \int_{\Omega_2} \left( (|\nabla u_1| + |\nabla u_2|)^{p-2} |\nabla u_1 - \nabla u_2|^2 g(x)^{-\varepsilon} \right)^{\frac{p-\varepsilon}{2}} \times (|\nabla u_1| + |\nabla u_2|)^{p-\varepsilon} \, d\mathbf{x}
\leq \left( \int_{\Omega_2} (|\nabla u_1| + |\nabla u_2|)^{p-2} |\nabla u_1 - \nabla u_2|^2 g(x)^{-\varepsilon} \, d\mathbf{x} \right)^{\frac{p-\varepsilon}{2}} \times \left( \int_{\Omega_2} (|\nabla u_1| + |\nabla u_2|)^{p-\varepsilon} \, d\mathbf{x} \right)^{\frac{p-\varepsilon}{2}}
\leq \left( \int_{\Omega_2} (|\nabla u_1| + |\nabla u_2|)^{p-2} |\nabla u_1 - \nabla u_2|^2 g(x)^{-\varepsilon} \, d\mathbf{x} \right)^{\frac{p-\varepsilon}{2}} \times \left( \int_{\Omega_2} (|\nabla u_1| + |\nabla u_2|)^{p-\varepsilon} \, d\mathbf{x} \right)^{\frac{p-\varepsilon}{2}}
\]
that is,
\[
\int_{\Omega_2} |\nabla u_1 - \nabla u_2|^{p-\varepsilon} \, d\mathbf{x}
\leq \left( \int_{\Omega_2} (|\nabla u_1| + |\nabla u_2|)^{p-2} |\nabla u_1 - \nabla u_2|^2 g(x)^{-\varepsilon} \, d\mathbf{x} \right)^{\frac{p-\varepsilon}{2}} \times \left( \int_{\Omega_2} (|\nabla u_1| + |\nabla u_2|)^{p-\varepsilon} \, d\mathbf{x} \right)^{\frac{p-\varepsilon}{2}}.
\]
Combining (3.5), (3.6), (3.7) and (3.11), there have
\[
\int_{\Omega_2} |\nabla u_1 - \nabla u_2|^{p-\varepsilon} \, d\mathbf{x}
\leq \left[ C \left( \frac{\varepsilon}{1-\varepsilon} \right)^{\frac{p-\varepsilon}{2}} \left( \int_{\Omega_2} (|\nabla u_1| + |\nabla u_2|)^{p-\varepsilon} \, d\mathbf{x} \right)^{\frac{p-\varepsilon}{2}} \left( \int_{\Omega_2} |\nabla u_1 - \nabla u_2|^{p-\varepsilon} \, d\mathbf{x} \right)^{\frac{1}{p-\varepsilon}} \right]
+ \varepsilon^{\varepsilon} C \left( \varepsilon'' \int_{\Omega_2} (|\nabla u_1| + |\nabla u_2|)^{p-\varepsilon} \, d\mathbf{x} + C(\varepsilon'')|\Omega_2| \right)^{\frac{p-\varepsilon}{2}} \times \left( \int_{\Omega_2} (|\nabla u_1| + |\nabla u_2|)^{p-\varepsilon} \, d\mathbf{x} \right)^{\frac{p-\varepsilon}{2}}.
\]
Similar to Case 1, we let \varepsilon \to 0, and Theorem 2.1 is proved.
非齐次椭圆方程很弱解的比较原理

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摘要: 在一些适当的假设条件下, 通过McShane扩张定理构造Lipschitz连续检验函数, 本文得到了拟线性椭圆方程(div A(x, u(x)) = f(x, u)很弱解的比较原理, 推广了齐次方程的相关结果。

关键词: 很弱解; 比较原理; Lipschitz连续检验函数; McShane扩张定理

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