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Stability Analysis of Indirect Adaptive Tracking Systems for Simple Linear Plants with Unknown Control Direction

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Abstract: The paper deals with the adaptive tracker for a class of first-order linear continuous-time plants, with different estimators and modified certainty-equivalence control law. We overcome the difficulty of the loss of stabilizability (or controllability) as the estimate of the plant gain is zero and correct a few substantive mistakes of the results from Middleton and Kokotovic's paper (1992) about the indirect adaptive regulation of the above-mentioned plant. In the constructed adaptive tracking systems, the explicit expression for the phase-plane trajectories or the explicit solution completely describing the nonlinear behavior of the resultant closed-loop systems is obtained, and some problems of the designed adaptive tracking systems, which due to a loss of stabilizability of the estimated model probably are analyzed. The paper also discusses the impact of these results for the indirect adaptive tracking case of higher order linear continuous-time plants. Meanwhile, we discuss the unknown both control direction and model parameters by making use of the similar pattern.

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1. Introduction

Let us consider a plant described by the following model:

$$A(s)y(t) = K_p B(s)u(t) \quad (K_p \neq 0), \tag{1.1}$$

where $u(t), y(t)$ are the plant input and output, respectively, K_p is the gain parameter of the plant, and s denotes the differential operator. The mimic polynomials $A(s), B(s)$ are coprime and of degrees n, m respectively, where $n \geqslant m$, and $B(s)$ is Hurwitz (i.e. $B(s) \neq 0, \forall\, \mathrm{Re}\, s \geqslant 0$).

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Assume the sign of the plant gain (i.e. the so-called control direction[2−3] of the plant) is unknown in this paper. However, it does usually need to assume that the sign of the plant gain is known both in direct adaptive control and indirect adaptive control[4]. In indirect adaptive control, the unknown control direction may cause the loss of stabilizability of the estimated plant model. In particular, the controllability is lost when the estimate of the unknown plant gain is zero. Therefore, the indirect adaptive control with unknown control direction is still a problem which has not been solved completely yet. How to use the different estimators together with a modified certainty-equivalence control law handling the passage of the gain estimate through zero? Which of their signals, if any, go unbounded? Do the causes for unboundedness suggest some "natural fixes", and should they be in the estimate algorithms or in the control law?

Middleton and Kokotovic[1] answered such basic questions on several extremely simple examples. Unfortunately, it didn't hit the point. We cite three examples in Section III of [1] as following:

1) For the control law (3.1) (i.e. $u(t) = -2y(t)/\hat{b}(t)$), if we choose $\hat{b}(0) < 0$ in the case of $b > 0$, or choose $\hat{b}(0) > 0$ in the case of $b < 0$, the initial control $u(0)$ of the resultant adaptive regulation system is positive feedback, which is a mistaken control direction. Its subsequent states will be divergent. Namely, it is impossible for them to keep boundedness properties. Even if $\mathrm{sign}(b) = 1$ is known, the control law (3.1) is still incorrect control strategy as selecting the initial estimate $\hat{b}(0) < 0$, which did not completely guarantee the boundedness properties of the controlled states of the resultant closed-loop system.

2) "It is clear from (3.1) that the control $u(t)$ is grown unboundedly as $\hat{b}(t)$ approaches zero for $y(t) \neq 0$". This is a obvious mistake too. Since when $\hat{b}(t) \to 0$, although $y(t) \neq 0$, it would converge to zero when $\hat{b}(t)$ converges to zero, this means $u(t)$ does not have to be unbounded. If $u(t)$ does increase unboundedly, then the boundedness properties of the controlled states of the resultantadaptive regulation system are no guarantee, which denies the main conclusion of themselves.

3) It was a crucial mistake that there is an unstable zero-pole cancellation when dividing the second equation in (3.6) by (3.3), thus the explicit expression of (3.8), the integral curve family defined by the equation (3.7) (see Fig.2 in [1]), is not the phase-diagram of the nonlinear state-space equations (3.2) and (3.3) (since the phase-plane trajectories of this system are undefined at $\hat{b}(t) = 0$), and the boundedness properties of η satisfying (3.7) does not mean the boundedness properties of y which satisfies (3.2) and (3.3). Therefore the proof of the boundedness properties of the controlled states is incorrect completely.

Even so, [1] is useful in certain sense. It contributes an important result that $\dot{\hat{b}}(t)$ has an unchanged sign which characterizes the significant property of related trajectories on the phase-plane. If we can choose $\mathrm{sign}(\hat{b}(0)) = \mathrm{sign}(b)$, which means the sign of the plant gain b needs to be known, then the boundedness properties could be guaranteed. However, in this case, there no longer exists problem of whether $\hat{b}(t)$ passes through zero. For a class of simple linear discrete-time plants, the correspondent adaptive regulation problem had solved completely due to [5].

Enlightened by [1,5], this paper, based on the same simple examples, provides a completely proper answer to the basic question above-mentioned and corrects a few mistakes in [1]. We consider the examples of the unstable first-order linear continuous-time plant with the known pole at the right half plane and the unknown both sign and value of the plant gain. Using three different estimate algorithms (i.e. the unnormalized gradient, the normalized gradient and the least squares) and combining with the modified certainty-equivalence pole-placement control law to handle the passage of the plant gain estimate through zero, we proceed on stabilizability analysis by finding the explicit solution to the nonlinear state space equationsor the explicit expression for the phase-plane trajectories, and discuss both boundedness properties and transient behavior of the controlled states of the resultantclosed-loop systems. This analysis provides us with a complete quantitative description of the boundedness properties and their limit properties.

2. The Formation of Adaptive Tracking System

I Plant model

Consider the first-order linear plant

$$\dot{y}(t) = y(t) + bu(t), \tag{2.1}$$

where $y(t)$ and $u(t)$ are the plant output and input, respectively, and b is an unknown nonzero constant parameter. We assume that an upper bound on the size of b is known, i.e.

$$|b| \leqslant \sigma. \tag{2.2}$$

While we assume that the sign of b is unknown, this is crucial.

Remark 2.1 In [1], the lower bound on the size of b is assumed, that is $\text{abs}(b) \geqslant \varepsilon > 0$. But it is only used to SCE control law (2.17) and DCE control law (2.18), not applied to the estimating algorithms (2.6)-(2.9) in [1]. In this paper, we don't need this restricted condition, but we need to know the upper bound of, which may reduce the scope of the selected initial value $\hat{b}(0)$ so as to move the closed-loop pole to the left plane (e.g., take $s = -1$) as soon as possible by adaptive tracking control.

II Parameter estimate algorithms

Assume that the bounded reference signal $r(t)$ is known, which will be the output $y(t)$'s tracking target of the resultant closed-loop system. Let $b(t)$ denote the estimate of unknown parameter b at time t. We define the prediction error $e(t)$ of the designed adaptive tracking system is

$$e(t) = \dot{y}(t) - \hat{\dot{y}}(t) = \dot{y}(t) - y(t) - b(t)u(t) = (b - b(t))\varphi(t). \tag{2.3}$$

The regressor in the above case is the scalar (the notation is also applied to the vector case)

$$\varphi(t) = u(t). \tag{2.4}$$

Remark 2.2 In order to simplify the calculations and avoid the trouble of constructing filters, we assume that the time derivative $\dot{y}(t)$ of the output $y(t)$ is available to the parameter estimators. But in order to preserve the realism of the feedback control problem, we also assume that the signal $\dot{y}(y)$ is not available to the feedback control itself.

The following three estimators to be analyzed are standard continuous-time parameter estimate algorithms for updating the parameter estimate[6−8]. Each of them defines the rate of change $\dot{b}(t)$ of $b(t)$ as a function of the regressor, prediction error, and a gain (either a fixed λ in the first two cases, or a variable $p(t)$ in the final case).

(a) UG-unnormalized gradient:

$$\dot{b}(t) = \lambda\varphi(t)e(t) = \lambda(b - b(t))u^2(t); \tag{2.5}$$

(b) NG-normalized gradient:

$$\dot{b}(t) = \frac{\lambda\varphi(t)e(t)}{1+\varphi^T(t)\varphi(t)} = \frac{\lambda(b-b(t))u^2(t)}{1+u^2(t)}; \tag{2.6}$$

(c) LS-least squares:

$$\dot{b}(t) = p(t)\varphi(t)e(t) = p(t)(b-b(t))u^2(t), \tag{2.7}$$

$$\dot{p}(t) = -p(t)\varphi^T(t)\varphi(t)p(t) = -p^2(t)u^2(t). \tag{2.8}$$

The initial estimate value in the three above algorithms should be selected within $[-\sigma, \sigma]$. Without loss of generality, we assume that λ and $p(0)$ are all selected to be positive values larger than 1.

Before we design the feedback control law and proceed with the analysis, let us introduce a property of the LS algorithm established in [1], which effectively eliminates one of the three state components in the analysis (but not in the implementation) of the LS-based adaptive tracking.

Lemma 2.1 For the plant (2.1), the least squares estimators (2.7) and (2.8) have the following property:

$$p(t)\tilde{b}(0) = p(0)\tilde{b}(t), \quad t \geqslant 0, \tag{2.9}$$

where $\tilde{b}(t) = b(t) - b, \ \forall t \geqslant 0$. In particular, when $\tilde{b}(0) \neq 0$, the equality (10) can be rewritten as

$$p(t) = p(0)\tilde{b}(t)/\tilde{b}(0), \quad t \geqslant 0. \tag{2.10}$$

Proof of the equality (2.9) had given in Lemma 2.1 of [1]. When $\tilde{b}(0) \neq 0$, the equality (2.10) has deduced immediately from the equality (2.9).

III Control law

Assumption H: The known reference signal $r(t)$ is continuous and bounded, and its time derivative $\dot{r}(t)$ is also piecewise continuous and bounded.

Let $e_r(t)$ denote the adaptive tracking error at time t, i.e., $e_r(t) = y(t) - r(t)$. We give a modified certainty equivalence pole-placement control strategy, attempting to place the closed-loop pole at the left plane (take $s = -1$):

$$u(t) = \begin{cases} 2g(t)e_r(t)/|b(t)|, & b(t) \neq 0, \\ 1/K(t), & b(t) = 0, \end{cases} \tag{2.11}$$

where $g(\cdot)$ is a special sign function, and it is defined as

$$g(t) = \begin{cases} -1, & t \in [t_0, t_0 + \Delta t], \\ \text{sign}\left\{ \frac{|e_r(t_1) - e_t(t_0)|}{\Delta t} - \max(|e_r(t_1) + e_r(t_0)|, |u(t_1) + u(t_0)|) \right\}, & \forall t \geqslant t_1 \end{cases} \tag{2.12}$$

and

$$t_1 = t_0 + \Delta t. \tag{2.13}$$

Here the initial time t_0 may be taken as zero, Δt denotes a fixed sampling period in the simulations or the implementations of the adaptive tracking control, $y(t_0)$ and $y(t_1)$ are the outputs of the plant (2.1) at the times t_0 and t_1, respectively, and $K(t)$ are a function about the gains λ or $p(t)$ for the estimators UG, NG or LS. $K(t) \equiv \sqrt{\lambda}$ for UG, $K(t) \equiv \sqrt{\lambda - 1}$ for NG and $K(t) \equiv \sqrt{p(t)}$ for LS.

The control law (2.11) clearly has a singularity when $b(t)$ is zero, while it is a removable singularity. Once $b(t)$ reaches zero at the time $t = t_z$, we can obtain from (2.11) and (2.5), (2.6) or (2.7) that for all $n \geqslant 1, t = t_z + n\Delta t, b(t) \equiv$the true value b (under the cases of used UG , NG or LS estimators) after implementing the control strategy $u(e_r(t), b(t)) = 1/K(t)$.

Remark 2.3 Although the control direction of the plant (2.1) is unknown, we may guess $\text{sign}(b) = 1$. If our guess is really right, the control strategy $u(0)$ defined in (2.11) starts to work whatever $b(0)$ is positive or negative, and can be defined from (2.12) that $g(t) = -1$ for all $t \geqslant t_0$. In this case, the implemented feedback control (2.11) for the plant (2.1) is always negative feedback. If our guess is wrong, then the sign of the plant gain b is negative. It shows that the initial control $u(0)$ of the adaptive tracking systems is positive feedback, which is a mistaken control direction. It can be calculated from (2.12) that $g(t) = 1$ for all $t \geqslant t_1$. Thus subsequent feedback control $u(e_r(t), b(t))$ for all $t \geqslant t_1$ will be corrected immediately as negative feedback. Therefore the control direction of $u(t), \forall t \geqslant t_1$, defined by (2.11) are always right, and $\text{sign}(bg(t)) = -1, \forall t \geqslant t_1$. The mistake of the control direction will at the most occur at the time t_0, which would not continually affect the succeeding controlled states.

3. Boundedness Properties of the Controlled States

In this section, we now proceed to analyze the three adaptive tracking systems with the modified certainty equivalence pole-placement control law (2.11) and with one estimate algorithm UG, and the rest two trackers (based on the estimate algorithms NG or LS) are omitted, since they follow a similar pattern. An important feature of these analyses is that we generate the explicit expression for the phase-plane trajectories or the explicit solution to the nonlinear state space equations. Therefore, they are able to completely describe the nonlinear behavior of the designed adaptive tracking systems.

Let us analyze the adaptive tracking systems with the unnormalized gradient estimation algorithm UG and the modified certainty equivalence pole-placement control law (2.11). For estimate algorithm UG, formula (2.11) may be rewritten as follows

$$u(e_r(t), b(t)) = \begin{cases} 2g(t)e_r(t)/|b(t)|, & b(t) \neq 0, \\ 1/\sqrt{\lambda}, & b(t) = 0. \end{cases} \tag{3.1}$$

The sign function $g(\cdot)$ in (3.1) is defined by (2.12). It is clear from (2.11) or (3.1) that the feedback control gain $2g(t)/|b(t)|$ grow unboundedly as $b(t)$ approaches zero. The sign function in (3.1) is defined by (2.12). It is clear from (2.11) or (3.1) that the feedback control gain grow unboundedly as approaches zero. The main question here is that whether such an unbounded feedbackcontrol gain will force the controlled state of the designed adaptive tracking system to grow unboundedly. Our answer is in the negative for such adaptive tracking system. Even when $b(t)$ passes through zero, the controlled states remain bounded and their transients are well behaved.

In this case, the state vector of the designed adaptive tracking system is composed of both the tracking error $e_r(t)$ of the resultant closed-loop system and the estimate $b(t)$ of the plant gain b. For convenience of both analysis and simulation, the true value of the plant gain b, unknown to the designer, is assumed to be

$$b = 1 \tag{3.2}$$

or

$$b = -1. \tag{3.3}$$

The other assumption for simplification of calculations is that the time derivative $\dot{e}_r(t)$ of the tracking error $e_r(t)$ is available to the parameter estimator, otherwise, the two filters will be introduced like what was done in [9-10]. To preserve the realism of the feedback control problem, we also assume that the signal $\dot{e}_r(t)$ is not available to the feedback control itself.

Using (2.1), (2.3)-(2.5), (3.1) and $e_r(t) = y(t) - r(t)$, we can write the nonlinear state-space equations as

$$\dot{e}_r(t) = e_r(t)(1 - 2g(t))/|b(t)|, \quad \forall t \geqslant 0, \tag{3.4}$$

$$\dot{b}_r(t) = 4\lambda e_r^2(t)(1 - b(t))/b^2(t), \quad \forall t \geqslant 0. \tag{3.5}$$

In the above (3.5), the true value is given by (3.2). If the true value is $b = -1$ given by (3.3), then the estimate formula (3.5) of b should be replaced by

$$\dot{b}_r(t) = -4\lambda e_r^2(t)(1 + b(t))/b^2(t), \quad \forall t \geqslant 0. \tag{3.6}$$

It is easily observed that $e_r(t) \equiv 0$ is an equilibrium manifold of (3.4) and (3.5) or is also the one of (3.4) and (3.6), and that the equilibria inside the segments $b(t) \in [0, 2]$ or $b(t) \in [-2, 0]$ are stable, while those outside segments are unstable. Possibly there exist two kinds of tendency of trajectories for the unstable cases: tend to an equilibrium or depart from all the equilibria.

It is clear from (3.5) that $b(0) = 1$ implies $b(t) = 1$ for all $t \geqslant 0$. Hence $b(t) \equiv 1 (\forall t \geqslant 0)$ is an invariant manifold of the differential equations (3.4) and (3.5), and its equilibrium solution under the initial condition of $b(0) = -1$ is

$$e_r(t) = \exp(-t)e_r(0), \quad b(t) = 1, \quad \forall t \geqslant 0. \tag{3.7}$$

It is also clear from (3.6) that $b(0) = -1$ implies $b(t) = -1$ for all $t \geqslant 0$. Hence $b(t) \equiv -1 (\forall t \geqslant 0)$ is also an invariant manifold of the differential equations (3.4) and (3.6), and its equilibrium solution of (3.4) and (3.6) under the initial condition of $b(0) = -1$ is

$$e_r(t) = \exp(-t)e_r(0), \quad b(t) = -1, \quad \forall t \geqslant 0. \tag{3.8}$$

These mean that if $t = 0$, the estimate $b(0)$ is correct, i.e. it equals the true value $b = 1$ or $b = -1$, then b(t) will continue to be correct for all $t \geqslant 0$.

The two invariant manifold pairs $\{e_r(t) \equiv 0, b(t) \equiv 1\}$ and $\{e_r(t) \equiv 0, b(t) \equiv -1\}$ of the two designed adaptive tracking systems divide respectively the corresponding phase-plane into four invariant quadrants. Further insight into the shape of the phase trajectories within each quadrant is provided by the fact that $b(t)$ defined by (3.5) or (3.6), respectively, is all of constantsign, i.e.

$$\text{sign}(\dot{b}(t)) = \text{sign}(1 - b(t)) = \text{sign}(1 - b(0)), \quad \forall t \geqslant 0 \tag{3.9}$$

or

$$\text{sign}(\dot{b}(t)) = -\text{sign}(1 + b(t)) = -\text{sign}(1 + b(0)), \quad \forall t \geqslant 0. \tag{3.10}$$

Thus, we know respectively from (3.9) and (3.10) that $b(t)$ is monotonically decreasing for $b(0) > 1$, or $b(0) > -1$, and monotonically increasing for $b(0) < 1$, or $b(0) < -1$. No matter what sign of the plant gain b is positive or negative, the equation (3.4) can be rewritten as follows

$$\dot{e}_r(t) = e_r(t)(1 - 2/|b(t)|), \quad \forall t \geqslant t_1, \tag{3.11}$$

where t_1 is given in (2.13). If $e_r(0) \neq 0$, we find the solution of (3.11) as

$$e_r(t) = e_r(t_1) + \exp(f(t_1, t)), \tag{3.12}$$

where

$$f(t_1, t) = \int_{t_1}^{t} (1 - 2/|b(\tau)|)\mathrm{d}\tau. \tag{3.13}$$

If $\mathrm{abs}(b(0)) \in (0, 2)$, then $\mathrm{abs}(b(t)) \in (0, 2), \forall t \geqslant 0$. It infers from (3.13) that $f(t_1, t) \leqslant 0, \forall t \geqslant t_1$ Hence the explicit solution $e_r(t)$ of (3.11) is of boundedness property, and furthermore

$$\frac{\mathrm{d}}{\mathrm{d}t}(e_r^2(t)) = 2e_r(t)\dot{e}_r(t) = e_r^2(t)(2 - 4/|b(t)|) < 0, \quad \forall |b(t)| \in (0, 2).$$

This implies that under the condition of $|b(t)| \in (0, 2), \forall t \geqslant 0, |b(t)|$ is strictly decreasing and so it is of limit zero, i.e. $\lim_{t\to\infty} e_r(t) = 0$. Therefore the differential equations (3.4), (3.5) and (3.4), (3.6) are all asymptotically stable.

If $|b(0)| = 2$, then both $\dot{e}_r(0) = 0$ and $\dot{b}(0) = 0$. This shows that they are critical points of the above-mentioned differential equations. Furthermore, both $e_r(t)$ and $b(t)$ are bounded. Also, the boundedness property of $y(t)$ is deduced from Assumption H about reference signal $r(t)$ and $e_r(t) = y(t) - r(t)$.

Remark 3.1 For the above-mentioned adaptive tracking system, we require $|b(0)| \in [0, 2]$ to ensure the boundedness properties and the convergency of $\{e_r(t), y(t), b(t); \forall t \geqslant 0\}$. This is a sufficient condition but not necessary. In fact, simulation examples in next section show that the designed adaptive tracking systems are asymptotically stable and the controlled states are bounded even if $|b(0)| > 2$. Also there exists a finite time $T \geqslant 0$ such that

$$|b(t)| \in [0, 2], \forall t \geqslant T.$$

Meanwhile, we also have

$$\lim_{x\to\infty} e_r(t) = 0.$$

In particular, if the true value is given by (3.2) or (3.3), and $b(0)$ is selected as $b(0) = -4$ or $b(0) = 4$, for the above three parameter estimate algorithms, then they can make $b(t)$ pass through zero without destroying the stability of the designed adaptive tracking systems. Also, two simulation examples in next section show that Assumption H about the time derivative $\dot{r}(t)$ of the reference signal $r(t)$(i.e. it is piecewise continuous and bounded) is needless too.

4. Simulations of Adaptive Tracking Systems

In Section 3, boundedness properties and convergency of the controlled states for the resultant adaptive tracking systems have been analyzed, which are mainly involved with limit property. Two simulation examples in this section are given to show the performance of the resultant adaptive tracking systems on the finite time zone.

Example 4.1 Simulation of adaptive tracking control for the plant (2.1) under the assumption (3.2) and two groups of suitable initial values.

First, we use the control law (3.1) with the update law (2.5) for the adaptive tracking simulation of the plant (2.1). In this example, tracked target of adaptive tracking control is the sinusoidal signal

$$r(t) = \sin(0.01\pi t), \quad \forall t \in [0, 80]. \tag{4.1}$$

Select the UG-estimating gains $\lambda_1 = 180$ and $\lambda_2 = 192$ for two different groups of initial value $\{y_1(0) = e_{1r}(0) = 0.06, b_1(0) = 4\}$ and $\{y_2(0) = e_{2r}(0) = -0.06, b_2(0) = -4\}$. Here we proceed to simulate, namely find digital solutions of the nonlinear differential equations (3.4) and (3.5) with some initial conditions chosen above. The simulation results are given in Fig. 4.1-4.4.

In Fig. 4.1, the outputs are $y_1(t)$ and $y_2(t)$ for the corresponding $\{\lambda_1, y_1(0), e_{1r}(0), b_1(0)\}$ and $\{\lambda_2, y_2(0), e_{2r}(0), b_2(0)\}$, respectively, where $y_1(t) - 0.4$ and $y_2(t) + 0.4$ denote the curves $y_1(t)$ and $y_2(t)$ downwards and upwards to move 0.4 unit altitude respectively. $u_1(t)$ and $u_2(t)$ in Fig. 4.2 denote the control quantities, respectively, corresponding to the two groups of the initial values. In Fig. 4.3, the adaptive tracking errors $e_{ir}(t)(i = 1, 2)$ also correspond to the two groups of the initial values, respectively. In Fig. 4.4, $b_1(t)$ and $b_2(t)$ are the curves of the estimate values of the plant gain, respectively, corresponding to the two groups of the initial values. In simulation, we set a fixed sampling period $\Delta t = 0.004$ seconds for the sampled-control system and the movement time 80 seconds for trajectories, but only 16 seconds picture is drawn in Fig. 4.4 because the parameter estimators are convergent so fast.

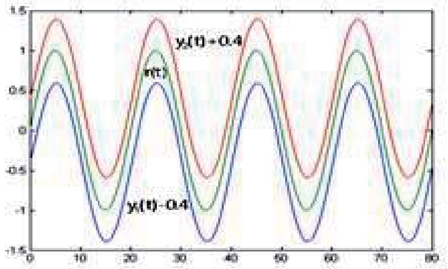

Fig. 4.1 Outputs $y_i(t)(i = 1, 2)$ tracking $r(t) = \sin(0.01\pi t)$ defined in (4.1)

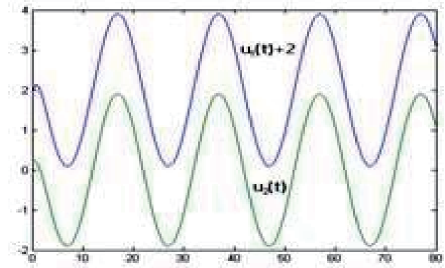

Fig. 4.2 Control quantities $u_i(t)(i = 1, 2)$ versus time t

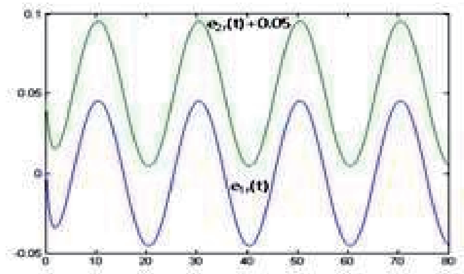

Fig. 4.3 Adaptive tracking errors $e_{ir}(t)(i = 1, 2)$ versus time t

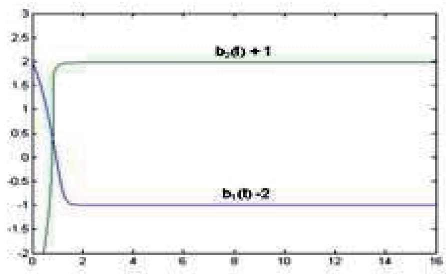

Fig. 4.4 UG-estimates $b_i(t)(i =, 1, 2)$ of the plant gain b versus time t

Example 4.2 Simulation of adaptive tracking control for the plant (2.1) under the assumption (3.3) and two groups of suitable initial values.

The adaptive tracking reference signal $r(t) = \sin(0, 01\pi t)$ in Example 4.1 is replaced to the following rectangular wave:

$$r(t) = \begin{cases} 1, & 40k \leqslant t < 40k + 20; \\ -1, & 40k + 20 \leqslant t < (k+1)40, \end{cases} \qquad k = 0, 1, 2, 3. \tag{4.2}$$

The function $r(t)$ is piecewise smooth and its derivative is the sum of four pairs of positive and negative impulse functions, i.e.,

$$\dot{r}(t) = \sum_{k=0}^{3}[\delta(t-40k)-\delta(t-40k-20)] \tag{4.3}$$

In this case, the equation (3.4) is unchanged and the equation (3.5) is replaced by the equation (3.6). We still use the control law (3.1) with the update law (2.5) for the adaptive tracking simulation of the plant (2.1). Two the UG-estimating gains $\lambda_i (i = 1, 2)$ and different group of initial values are selected the same as Example 4.1. The simulation results are given in Fig. 4.5-4.8.

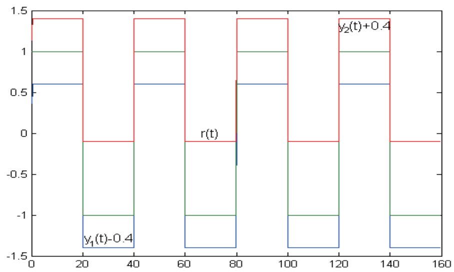

Fig. 4.5 Outputs $y_i(t)(i = 1, 2)$ tracking $r(t)$ defined in (4.2)

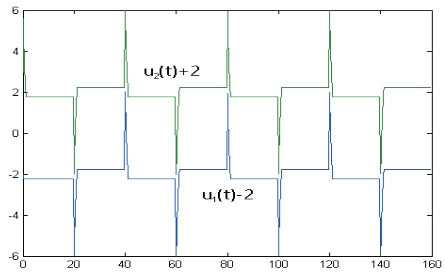

Fig. 4.6 Control quantities $u_i(t)(i = 1, 2)$ versus time t

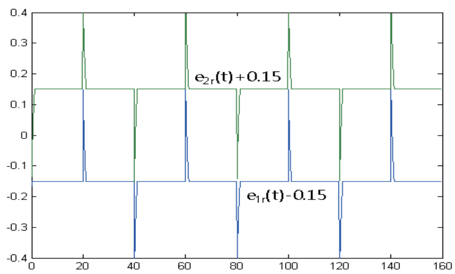

Fig. 4.7 Adaptive tracking errors $e_{ir}(t)(i = 1, 2)$ versus time t

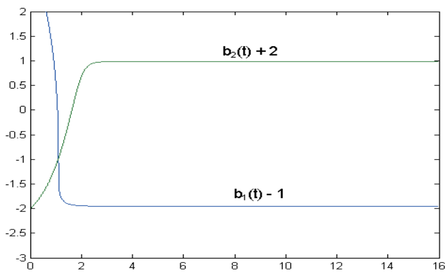

Fig. 4.8 UG-estimates $b_i(t)(i = 1, 2)$ of the plant gain b versus time t

We also use the control law (2.11) with the update estimate (2.1) or (2.7) for the adaptive tracking simulations of the plant (2.1). The simulation results are similar to the corresponding results of the UG-based adaptive tracking simulation. Therefore we omitted their simulation pictures. It should be noted when the simulation is going on with the LS estimate algorithm, and initial value $p(0)$ should be selected much larger than the values of two parameter estimate gains λ_1 and λ_2. Otherwise, the excitation signal could be too weak hard to guarantee the convergence rate of the parameter estimate $b(t)$.

5. Conclusions

Under the conditions of unknown both sign and value of the plant gain, although the sign of the initial estimates of the plant gain $b_i(0)(i = 1, 2)$ are probably incorrect, the three parameter estimate algorithms respectively combining the modified certainty equivalence control law are all able to drive that the estimators $b(t)$ pass through zero and quickly converge to the true value of b. Thus, the controlled states of the designed adaptive tracking system are bounded, and the tracking errors $e_r(t) \to 0 (t \to \infty)$. At the same time, their transients are well behaved.

For the higher order plant (1.1) with the unknown control direction, we can generalize the modified certainty-equivalence control strategy (2.11) to the plant (1.1), and attempt to place all the closed-loop poles at $s = -1$. Under this case, $b(t)$ in the control strategy (2.11) is replaced by $K_p(t)$. We can still use three estimate algorithms UG, NG and LS to estimate K_p and all the unknown parameters of both $A(s)$ and $B(s)$ in (1.1). The indirect adaptive control problem with unknown control direction can be solved as long as we select properly the estimate gain λ for UG and NG, in particular, select carefully the initial value $p(0)$ of the estimate gain for LS.

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未知控制方向的简单线性对象间接自适应跟踪系统的稳定性分析

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摘要: 本文研究一类一阶线性连续时间对象的自适应跟踪器, 并给出不同估计量和修正确定性等价控制律. 我们克服了确定损失的困难(或控制)的系统增益的估计值是零, 纠正了米德尔顿和Kokotovic的文章(1992)对上述对象的间接自适应调节的一些实质性的错误的结果. 在构建的自适应跟踪系统中, 得到相平面轨迹或完全描述闭环系统的非线性行为的显式解的显式表达式, 并对所设计的自适应跟踪系统可能由损失估计模型的稳定性所带来问题进行分析. 讨论了这些结果对高阶线性连续时间对象的间接自适应跟踪情况的影响. 同时通过类似的模式讨论了未知的控制方向和模型参数.

关键词: 间接自适应跟踪; 稳定性分析; 有界性; 控制方向; 非线性行为