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On $(n-1,1)$ Conjugate Boundary Value Problems with Dependence on Fully Nonlinearity

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Abstract: In this paper, a fixed point theorem in a cone and some inequalities of the associated Green's function are applied to obtain the existence of positive solutions of $(n-1,1)$ conjugate boundary value problems with dependence on all lower order derivatives.
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1. Introduction

In recent years, there has been much attention focused on questions of positive solutions of conjugate boundary value problems for nonlinear ordinary differential equations, difference equations, and functional differential equations without dependence on the first order derivative[1,3−6,8,10−11]. It is well known that the famous Krasnosel'skii's fixed point theorem in a cone[5], as well as Leggett-Williams fixed point theorem[6], plays an extremely important role in above works.

However, all the above works were done under the assumption that the derivatives are not involved explicitly in the nonlinear term. For the lower order derivatives which are involved explicitly in the nonlinear term, the study is few[2,9]. In this paper, via a generalization of Krasnosel'skii's fixed point theorem in a cone and some inequalities of the associated Green's function for the associated problem, we show the existence of positive solutions for the $(n-1,1)$ conjugate boundary value problem

$$u^{(n)}(x)+f(x,u(x),u'(x),\cdots,u^{(n-1)}(x))=0,\quad 0<x<1, \tag{1.1}$$

$$u^{(k)}(0)=u(1)=0,\quad 0\le k\le n-2, \tag{1.2}$$

where $n\ge 2, f:[0,1]\times[0,\infty)\times\mathbb{R}^{n-1}\to[0,\infty)$ is continuous. For $n=2$, LI[7] studied the problem (1.1)-(1.2) under the conditions that the nonlinearity $f(x,u,v)$ may be superlinear

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or sublinear growth on u and v. The superlinear and sublinear growth of the nonlinearity f are described by inequality conditions instead of the usual upper and lower limits conditions. The discussion is based on the fixed point index theory on cones.

As it is pointed out in [4], for $n = 2$, positive solutions of Problem (1.1)-(1.2) are concave and this concavity was useful in define a cone on which a positive operator was defined, to which a fixed point theorem due to Krasnosel'skii was then applied to yield positive solutions. For the general problem considered in this paper, the corresponding property was obtained by Eloe et al.[4] with the classical maximum principle. Readers may find that the property is crucial in defining an appropriate cone in this paper such that a generalization of Krasnosel′skii's fixed point theorem due to BAI and GE[2] can be used to obtain positive solutions.

2. Preliminaries and Lemmas

Let X be a Banach space and $P \subset X$ is a cone. Suppose $\alpha, \beta : X \to \mathbb{R}^+$ are two continuous nonnegative functionals satisfying

$$\alpha(\lambda u) \le |\lambda|\alpha(u), \quad \beta(\lambda u) \le |\lambda|\beta(u), \quad \text{for } u \in X,\ \lambda \in [0,1], \tag{2.1}$$

and

$$M_1 \max\{\alpha(u), \beta(u)\} \le \|u\| \le M_2 \max\{\alpha(u), \beta(u)\}, \quad \text{for } u \in X, \tag{2.2}$$

where M_1, M_2 are two positive constants.

Lemma 2.1[2] Let $r_2 > r_1 > 0, L_2 > L_1 > 0$ be constants and

$$\Omega_i = \{u \in X \mid \alpha(u) < r_i, \beta(u) < L_i\}, \quad i = 1, 2$$

are two open subsets in X such that $\theta \in \Omega_1 \subset \overline{\Omega}_1 \subset \Omega_2$. In addition, let

$$C_i = \{u \in X \mid \alpha(u) = r_i, \beta(u) \le L_i\}, \quad i = 1, 2;$$
$$D_i = \{u \in X \mid \alpha(u) \le r_i, \beta(u) = L_i\}, \quad i = 1, 2.$$

Assume $T : P \to P$ is a completely continuous operator satisfying

(S$_1$) $\alpha(Tu) \le r_1, u \in C_1 \cap P;\ \ \beta(Tu) \le L_1, u \in D_1 \cap P;$
$\alpha(Tu) \ge r_2, u \in C_2 \cap P;\ \ \beta(Tu) \ge L_2, u \in D_2 \cap P;$

or

(S$_2$) $\alpha(Tu) \ge r_1, u \in C_1 \cap P;\ \ \beta(Tu) \ge L_1, u \in D_1 \cap P;$
$\alpha(Tu) \le r_2, u \in C_2 \cap P;\ \ \beta(Tu) \le L_2, u \in D_2 \cap P;$

then T has at least one fixed point in $(\overline{\Omega}_2 \setminus \Omega_1) \cap P$.

Lemma 2.2[4] Let $n \ge 2$ and $u \in C^n[0,1]$ satisfy

$$-u^{(n)}(x) \ge 0, \quad 0 \le x \le 1,$$
$$u^{(k)}(0) \ge 0, \quad u(1) \ge 0, \quad 0 \le k \le n-2.$$

Then

$$u(x) \ge \frac{1}{4^{n-1}} \max_{x\in[0,1]} |u(x)|, \quad \frac{1}{4} \le x \le \frac{3}{4}. \tag{2.3}$$

Lemma 2.3[4] Let $G(x,s)$ be Green's function for the boundary value problem

$$-u^{(n)}(x) = 0, \quad 0 \le x \le 1, \tag{2.4}$$

$$u^{(k)}(0) = u(1) = 0, \quad 0 \le k \le n-2. \tag{2.5}$$

Then, for each $0 < s < 1$,

$$G(x,s) \ge \frac{1}{4^{n-1}} \max_{0\le x\le 1} G(x,s), \quad \frac{1}{4} \le x \le \frac{3}{4}. \tag{2.6}$$

3. Existence Results of Positive Solutions

In this section, we obtain positive solutions of Problem (1.1)-(1.2) by the use of Lemmas 2.1-2.3.

Problem (1.1), (1.2) has a solution $u = u(x)$ if and only if u solves the operator equation

$$u(x) = (Tu)(x) := \int_0^1 G(x,s) f(s, u(s), u'(s), \cdots, u^{(n-1)}(s)) \mathrm{d}s, \ 0 \le x \le 1,$$

where $G(x,s) \ge 0$ is Green's function for the boundary value problem (2.4)-(2.5)

$$G(x,s) = \begin{cases} \frac{x^{n-1}(1-s)^{n-1} - (x-s)^{n-1}}{(n-1)!} := Q(x,s), & 0 \le x \le s \le 1; \\ \frac{x^{n-1}(1-s)^{n-1}}{(n-1)!} := R(x,s), & 0 \le s \le x \le 1. \end{cases}$$

It is clear that while the $(n-2)$-order derivative of function $G(x,s)$ on x is continuous in $[0,1] \times [0,1]$, the $(n-1)$-order derivative of function $G(x,s)$ on x exists in $[0,1] \times [0,1]$ but it is not continuous for $x = s$.

Let $X = \{u \in C^{n-1}[0,1] \mid u^{(k)}(0) = u(1) = 0, \ \ 0 \le k \le n-2\}$ with

$$\|u\| = \max_{0 \le k \le n-1} \left\{ \max_{0 \le x \le 1} |u^{(k)}(x)| \right\},$$

then $(X, \|\cdot\|)$ is a Banach space. We seek solutions of (1.1), (1.2) that lie in a cone, P, defined by

$$P = \left\{ u \in X \mid u(x) \ge 0, \ \text{and} \ \min_{\frac{1}{4} \le x \le \frac{3}{4}} u(x) \ge \frac{1}{4^{n-1}} \max_{0 \le x \le 1} |u(x)| \right\}.$$

Define functionals

$$\alpha(u) = \max_{0 \le x \le 1} |u(x)|, \ \beta(u) = \max_{0 \le x \le 1} |u^{(n-1)}(x)|, \ \text{for } u \in X,$$

then $\alpha, \beta : X \to \mathbb{R}^+$ are continuous functionals. Taking into account that $u^{(k)}(0) = 0, \ k = 0, 1, \cdots, n-2$, it is clear that for $u \in X$,

$$\max_{0 \le x \le 1} |u^{(k-1)}(x)| \le \max_{0 \le x \le 1} |u^{(k)}(x)|.$$

So, $\|u\| = \max\{\alpha(u), \beta(u)\}$, the assumptions (2.1), (2.2) hold.

Denote

$$M = \max_{0 \le x \le 1} \int_0^1 G(x,s) \mathrm{d}s, \quad N = \max_{0 \le x \le 1} \int_{\frac{1}{4}}^{\frac{3}{4}} G(x,s) \mathrm{d}s,$$

$$A = \int_0^1 \left[\frac{\partial^{n-1}}{\partial x^{n-1}} R(x,s) \right]_{x=1} \mathrm{d}s = \int_0^1 (1-s)^{n-1} \mathrm{d}s = \frac{1}{n},$$

$$\overline{A} = \int_{\frac{1}{4}}^{\frac{3}{4}} \left[\frac{\partial^{n-1}}{\partial x^{n-1}} R(x,s) \right]_{x=1} \mathrm{d}s = \frac{3^n - 1}{n 4^n},$$

$$B = \max_{0 \le x \le 1} \left\{ \int_0^x \left| \frac{\partial^{n-1}}{\partial x^{n-1}} R(x,s) \right| \mathrm{d}s + \int_x^1 \left| \frac{\partial^{n-1}}{\partial x^{n-1}} Q(x,s) \right| \mathrm{d}s \right\}.$$

It is well known that $T : P \to P$ is completely continuous. In fact, if $u \in P$, then the continuity of f and the property of $G(x,s)$ yields $Tu \in C^{n-1}[0,1]$. By (2.6),

$$(Tu)(x) = \int_0^1 G(x,s) f(s, u(s), u'(s), \cdots, u^{(n-1)}(s)) \mathrm{d}s$$
$$\le \int_0^1 \max_{0 \le x \le 1} G(x,s) f(s, u(s), u'(s), \cdots, u^{(n-1)}(s)) \mathrm{d}s,$$

so that

$$\alpha(Tu)=\max_{0\le x\le 1}(Tu)(x)\le\int_0^1\max_{0\le x\le 1}G(x,s)f(s,u(s),u'(s)\cdots,u^{(n-1)}(s))\mathrm{d}s.$$

Combine with (2.3), one has

$$\begin{aligned}\min_{\frac14\le x\le\frac34}(Tu)(x)&=\min_{\frac14\le x\le\frac34}\int_0^1 G(x,s)f(s,u(s),u'(s),\cdots,u^{(n-1)}(s))\mathrm{d}s\\&\ge\frac{1}{4^{n-1}}\int_0^1\max_{0\le x\le 1}G(x,s)f(s,u(s),u'(s),\cdots,u^{(n-1)}(s))\mathrm{d}s\\&\ge\frac{1}{4^{n-1}}\alpha(Tu).\end{aligned}$$

Also, from the positivity of $G(x,s)$, for $u\in P$, there holds $(Tu)(x)\ge 0, 0\le x\le 1$. The definition of $G(x,s)$ gives Tu satisfies boundary conditions. Consequently, $T:P\to P$. Further, standard arguments yields that T is completely continuous.

Theorem 3.1 Let $f:[0,1]\times[0,\infty)\times\mathbb{R}^{n-1}\to[0,\infty)$ be continuous. Suppose there exist four constants $r_2>r_1>0,\ L_2>L_1>0$ such that $\max\{\frac{r_1}{M},\frac{L_1}{A}\}\le\min\{\frac{r_2}{M},\frac{L_2}{B}\}$, and the following assumptions hold:

(A_1) $f(x,u,v)\ge\max\{\frac{r_1}{M},\frac{L_1}{A}\}$, for $(x,u,v)\in[0,1]\times[0,r_1]\times[-L_1,L_1]^{n-1}$;

(A_2) $f(x,u,v)\le\min\{\frac{r_2}{M},\frac{L_2}{B}\}$, for $(x,u,v)\in[0,1]\times[0,r_2]\times[-L_2,L_2]^{n-1}$,

where $v=\{v_1,v_2,\cdots,v_{n-1}\}$. Then Problem (1.1)-(1.2) has at least one positive solution $u(x)$ such that

$$r_1\le\max_{0\le x\le 1}u(x)\le r_2,\qquad\text{or}\qquad L_1\le\max_{0\le x\le 1}|u^{(k)}(x)|\le L_2,\ k=1,2,\cdots,n-1.$$

Proof Let

$$\Omega_i=\{u\in X\mid\alpha(u)<r_i,\beta(u)<L_i\},\quad i=1,2,$$

be two bounded open subsets in X. In addition, let

$$C_i=\{u\in X\mid\alpha(u)=r_i,\beta(u)\le L_i\},\quad i=1,2;$$
$$D_i=\{u\in X\mid\alpha(u)\le r_i,\beta(u)=L_i\},\quad i=1,2.$$

For $u\in C_1\cap P$, by (A_1), there is

$$\begin{aligned}\alpha(Tu)&=\max_{x\in[0,1]}\left|\int_0^1 G(x,s)f(s,u(s),u'(s),\cdots,u^{(n-1)}(s))\mathrm{d}s\right|\\&\ge\frac{r_1}{M}\max_{x\in[0,1]}\left|\int_0^1 G(x,s)\mathrm{d}s\right|=r_1.\end{aligned}$$

According to the definition of T, we have

$$\begin{aligned}(Tu)^{(n-1)}(x)=&\int_0^x\frac{\partial^{n-1}}{\partial x^{n-1}}R(x,s)f(s,u(s),u'(s),\cdots,u^{(n-1)}(s))\mathrm{d}s\\&+\int_x^1\frac{\partial^{n-1}}{\partial x^{n-1}}Q(x,s)f(s,u(s),u'(s),\cdots,u^{(n-1)}(s))\mathrm{d}s,\ 0\le x\le 1.\end{aligned}$$

Combine with (A_1) and $f\ge 0$, for $u\in D_1\cap P$, there is

$$\begin{aligned}\beta(Tu)&=\max_{x\in[0,1]}|(Tu)^{(n-1)}(x)|\ge|(Tu)^{(n-1)}(1)|\\&=\int_0^1\left[\frac{\partial^{n-1}}{\partial x^{n-1}}R(x,s)\right]_{x=1}f(s,u(s),u'(s),\cdots,u^{(n-1)}(s))\mathrm{d}s\\&\ge\frac{L_1}{A}\int_0^1\left[\frac{\partial^{n-1}}{\partial x^{n-1}}R(x,s)\right]_{x=1}\mathrm{d}s=L_1.\end{aligned}$$

For $u \in C_2 \cap P$, by (A_2), there is

$$\alpha(Tu) = \max_{x\in[0,1]} \left| \int_0^1 G(x,s) f(s,u(s),u'(s),\cdots,u^{(n-1)}(s))\mathrm{d}s \right|$$
$$\leq \max_{x\in[0,1]} \int_0^1 \frac{r_2}{M} G(x,s)\mathrm{d}s = \frac{r_2}{M} \max_{x\in[0,1]} \int_0^1 G(x,s)\mathrm{d}s = r_2.$$

For $u \in D_2 \cap P$, by (A_2), there is

$$\beta(Tu) = \max_{x\in[0,1]} |(Tu)^{(n-1)}(x)|$$
$$= \max_{x\in[0,1]} \left| \int_0^x \frac{\partial^{n-1}}{\partial x^{n-1}} R(x,s) f(s,u(s),u'(s),\cdots,u^{(n-1)}(s))\mathrm{d}s + \int_x^1 \frac{\partial^{n-1}}{\partial x^{n-1}} Q(x,s) f(s,u(s),u'(s),\cdots,u^{(n-1)}(s))\mathrm{d}s \right|$$
$$\leq \max_{x\in[0,1]} \left\{ \int_0^x \left| \frac{\partial^{n-1}}{\partial x^{n-1}} R(x,s) \right| f(s,u(s),u'(s),\cdots,u^{(n-1)}(s))\mathrm{d}s + \int_x^1 \left| \frac{\partial^{n-1}}{\partial x^{n-1}} Q(x,s) \right| f(s,u(s),u'(s),\cdots,u^{(n-1)}(s))\mathrm{d}s \right\}$$
$$\leq \frac{L_2}{B} \cdot B = L_2.$$

Now, Lemma 2.1 implies there is $u \in (\overline{\Omega}_2 \setminus \Omega_1) \cap P$ such that $u = Tu$, namely, Problem (1.1)-(1.2) has at least one positive solution $u(x)$ such that

$$r_1 \leq \alpha(u) \leq r_2, \quad \text{or} \quad L_1 \leq \beta(u) \leq L_2,$$

i.e.,

$$r_1 \leq \max_{0\leq x\leq 1} u(x) \leq r_2, \quad \text{or} \quad L_1 \leq \max_{0\leq x\leq 1} |u^{(k)}(x)| \leq L_2,\ k = 1,2,\cdots,n-1.$$

The proof is completed.

Theorem 3.2 Let $f : [0,1] \times [0,\infty) \times \mathbb{R}^{n-1} \to [0,\infty)$ be continuous. Suppose there exist four constants $0 < r_1 < r_2,\ 0 < L_1 < L_2$ such that $\max\{\frac{r_1}{N}, \frac{L_1}{A}\} \leq \min\{\frac{r_2}{M}, \frac{L_2}{B}\}$, and the following assumptions hold:

(A_3) $f(x,u,v) \geq \frac{r_1}{N}$, for $(x,u,v) \in [\frac{1}{4},\frac{3}{4}] \times [\frac{r_1}{4^{n-1}}, r_1] \times [-L_1,L_1]^{n-1}$;

(A_4) $f(x,u,v) \geq \frac{L_1}{A}$, for $(x,u,v) \in [\frac{1}{4},\frac{3}{4}] \times [0, r_1] \times [-L_1,L_1]^{n-1}$;

(A_5) $f(x,u,v) \leq \min\{\frac{r_2}{M}, \frac{L_2}{B}\}$, for $(x,u,v) \in [0,1] \times [0, r_2] \times [-L_2,L_2]^{n-1}$,

where $v = \{v_1, v_2, \cdots, v_{n-1}\}$. Then Problem (1.1)-(1.2) has at least one positive solution $u(x)$ such that

$$r_1 \leq \max_{0\leq x\leq 1} u(x) \leq r_2, \quad \text{or} \quad L_1 \leq \max_{0\leq x\leq 1} |u^{(k)}(x)| \leq L_2,\ k = 1,2,\cdots,n-1.$$

Proof We just need to notice the following difference to Theorem 3.1.

For $u \in C_1 \cap P$, the definition of P implies that

$$u(x) \geq \frac{1}{4^{n-1}} \alpha(u) = \frac{r_1}{4^{n-1}}, \quad \text{for } x \in \left[\frac{1}{4}, \frac{3}{4}\right].$$

By (A_3), there is

$$\alpha(Tu) = \max_{x\in[0,1]} \left| \int_0^1 G(x,s) f(s,u(s),u'(s),\cdots,u^{(n-1)}(s))\mathrm{d}s \right|$$
$$\geq \max_{x\in[0,1]} \left| \int_{\frac{1}{4}}^{\frac{3}{4}} G(x,s) f(s,u(s),u'(s),\cdots,u^{(n-1)}(s))\mathrm{d}s \right|$$

$$\geq \max_{x\in[0,1]}\left|\int_{\frac{1}{4}}^{\frac{3}{4}}\frac{r_1}{N}G(x,s)\mathrm{d}s\right| = \frac{r_1}{N}\max_{x\in[0,1]}\left|\int_{\frac{1}{4}}^{\frac{3}{4}}G(x,s)\mathrm{d}s\right| = r_1.$$

For $u\in D_1\cap P$, by (A_4), there is

$$\beta(Tu)\geq|(Tu)^{(n-1)}(1)| = \int_0^1\left[\frac{\partial^{n-1}}{\partial x^{n-1}}R(x,s)\right]_{x=1}f(s,u(s),u'(s),\cdots,u^{(n-1)}(s))\mathrm{d}s$$

$$\geq\int_h^{1-h}\left[\frac{\partial^{n-1}}{\partial x^{n-1}}R(x,s)\right]_{x=1}f(s,u(s),u'(s),\cdots,u^{(n-1)}(s))\mathrm{d}s\geq\frac{L_1}{\overline{A}}\cdot\overline{A}=L_1.$$

The rest of proof is similar to Theorem 3.1 and the proof is completed.

4. Examples

Example 4.1 Consider the following boundary value problem:

$$u^{(4)}(x)+f(x,u(x),u'(x),u''(x),u'''(x))=0,\quad 0<x<1, \tag{4.1}$$

$$u''(0)=u'(0)=u(0)=u(1)=0, \tag{4.2}$$

where

$$f(x,u,v_1,v_2,v_3)=1+|u|^{\frac{1}{2}}+\sum_{k=1}^{3}|v_k|^{\frac{1}{2}}.$$

Choose

$$r_1=\frac{1}{96},\quad r_2=100,\quad L_1=\frac{1}{8},\quad L_2=400.$$

We note

$$\max\left\{\frac{r_1}{M},\frac{L_1}{A}\right\}=\frac{1}{2},\quad \min\left\{\frac{r_2}{M},\frac{L_2}{B}\right\}=1600,\quad \frac{L_1}{A}=\frac{1}{2},\quad \frac{L_2}{B}=1600.$$

Consequently, there hold

$$f(x,u,v_1,v_2,v_3)\geq 1\geq\frac{1}{2}\ \text{ for } 0\leq x\leq 1,\quad 0\leq u\leq\frac{1}{96},\quad -\frac{1}{8}\leq v_k\leq\frac{1}{8},\ k=1,2,3;$$

$$f(x,u,v_1,v_2,v_3)\leq 71\leq 6000,\quad \text{for } 0\leq x\leq 1,\quad 0\leq u\leq 100,\quad -400\leq v_k\leq 400,\ k=1,2,3.$$

Then all assumptions of Theorem 3.1 hold. Thus with Theorem 3.1, the problem (4.1)-(4.2) has at least one positive solution $u(x)$ such that

$$\frac{1}{96}\leq\max_{0\leq x\leq 1}u(x)\leq 100,\quad\text{or}\quad\frac{1}{8}\leq\max_{0\leq x\leq 1}|u^{(k)}(x)|\leq 400,\quad k=1,2,3.$$

Example 4.2 Consider the following boundary value problem:

$$u'''(x)+f(x,u(x),u'(x),u''(x))=0,\quad 0<x<1, \tag{4.3}$$

$$u''(0)=u'(0)=u(0)=u(1)=0, \tag{4.4}$$

where

$$f(x,u,v_1,v_2)=8x+40u^{\frac{1}{2}}+\sum_{k=1}^{2}|v_k|^{\frac{1}{2}}.$$

Choose

$$r_1=\frac{1}{2},\quad r_2=4000,\quad L_1=\frac{1}{4},\quad L_2=2000.$$

We note

$$\max\left\{\frac{r_1}{N},\frac{L_1}{\overline{A}}\right\}=\frac{96}{13},\quad \min\left\{\frac{r_2}{M},\frac{L_2}{B}\right\}=6000,\quad \frac{r_1}{N}=\frac{96}{13},\quad \frac{L_1}{\overline{A}}=\frac{24}{13}.$$

Consequently, there hold

$$f(x,u,v_1,v_2)\geq 9\geq\frac{96}{13}\approx 7.385,\quad\text{for }\frac{1}{4}\leq x\leq\frac{3}{4},\quad\frac{1}{32}\leq u\leq\frac{1}{2},\quad -\frac{1}{4}\leq v_k\leq\frac{1}{4},\ k=1,2;$$

$$f(x,u,v_1,v_2)\ge 2\ge \frac{24}{13}\approx 1.846,\quad \text{for } \frac{1}{4}\le x\le \frac{3}{4},\ \ 0\le u\le \frac{1}{2},\ \ -\frac{1}{4}\le v_k\le \frac{1}{4},\ k=1,2;$$

$$f(x,u,v_1,v_2)\le 2630\le 6000,\quad \text{for } 0\le x\le 1,\ \ 0\le u\le 4000,\ \ -2000\le v_k\le 2000,\ k=1,2.$$

Then all assumptions of Theorem 3.2 hold.

Thus with Theorem 3.2, the problem (4.3)-(4.4) has at least one positive solution $u(x)$ such that

$$\frac{1}{2}\le \max_{0\le x\le 1} u(x)\le 4000,\quad \text{or}\quad \frac{1}{4}\le \max_{0\le x\le 1}|u^{(k)}(x)|\le 2000,\quad k=1,2.$$

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完全非线性依赖的$(n-1,1)$共轭边值问题

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摘要：本文利用不动点定理和一些相关格林函数的不等式得到一个依赖于所有低阶导数的$(n-1,1)$共轭边值问题正解的存在性.

关键词：共轭边值问题; 锥上不动点定理; 正解